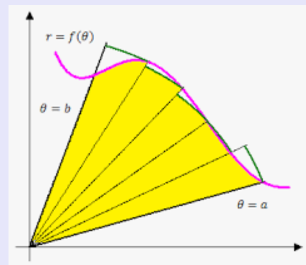
Calculus II

Lecture 7



Feb 19-8:47 AM

Class QZ 6

1) Find $\lim_{x\to\infty} \frac{\sqrt{x}}{x^2} = \frac{\sqrt{\infty}}{\infty^2} = \frac{\infty}{\infty}$ I.F.

$$= \lim_{x\to\infty} \frac{\frac{1}{2\sqrt{x}}}{2x} = \lim_{x\to\infty} \frac{1}{4x\sqrt{x}} = \frac{1}{\infty} = \boxed{0}$$

2) Find $\lim_{x\to\infty} \left[\ln x - \ln(x+1)\right] = \infty - \infty$

$$\lim_{x\to\infty} \frac{x}{x+1} = 1$$

$$= \lim_{x\to\infty} \ln \frac{x}{x+1}$$

$$= \lim_{x\to\infty} \ln 1 = \boxed{0}$$

Jun 20-6:40 AM

7.1 Integration by Parts

$\int x\,dx = \frac{1}{2}x^2 + C$, $\int e^x dx = e^x + C$

what about $\int xe^x dx$?

$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$

$u=$ $dv=$

$du=$ $v=$

$\int f(x)g(x)\,dx \neq \int f(x)dx \cdot \int g(x)dx$

$uv - \int v\,du$ Integration by Parts.

$\int xe^x dx$

$u = x$ $dv = e^x dx$

$du = dx$ $v = \int e^x dx = e^x$

$= xe^x - \int e^x dx = \boxed{xe^x - e^x + C}$

Jun 20-8:15 AM

$\int x^2 \sin x\, dx$

$u = x^2$ $dv = \sin x\, dx$ $= uv - \int v\,du$

$du = 2x\,dx$ $v = -\cos x$ $= -x^2\cos x - \int -\cos x \cdot 2x\,dx$

$= -x^2\cos x + 2\int x\cos x\,dx$

$= -x^2\cos x + 2\left[x\sin x - \int \sin x\,dx\right]$

$u = x$ $dv = \cos x\,dx$

$du = dx$ $v = \sin x$

$= -x^2\cos x + 2x\sin x - 2(-\cos x) + C$

$= -x^2\cos x + 2x\sin x + 2\cos x + C$

Jun 20-8:22 AM

Find $\int (\ln x)^2\,dx$ $u=(\ln x)^2$ $dv=dx$

$uv-\int v\,du$ $du=2(\ln x)^1\cdot\frac{1}{x}$ $v=x$

$= x(\ln x)^2-\int \cancel{x}\cdot 2\ln x\cdot\frac{1}{\cancel{x}}\,dx$

$= x(\ln x)^2-2\int \ln x\,dx$ $u=\ln x$ $dv=dx$

$du=\frac{1}{x}dx$ $v=x$

$= x(\ln x)^2-2\left[x\ln x-\int \cancel{x}\cdot\frac{1}{\cancel{x}}\,dx\right]$

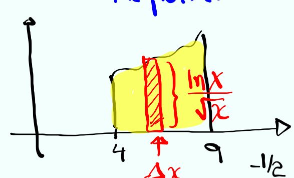
$= \boxed{x(\ln x)^2-2x\ln x+2x+C}$

Jun 20-8:29 AM

Find the area below $f(x)=\frac{\ln x}{\sqrt{x}}$, above x-axis from $x=4$ to $x=9$. Drawing Required

$x=4 \rightarrow y=\frac{\ln 4}{2}>0$

$x=9 \rightarrow y=\frac{\ln 9}{3}>0$



$A=\int_4^9 \frac{\ln x}{\sqrt{x}}\,dx$ $u=\ln x$ $dv=\frac{1}{\sqrt{x}}dx \rightarrow x^{-1/2}$

$v=\frac{x^{1/2}}{1/2}=2\sqrt{x}$

$=2\sqrt{x}\ln x-\int 2\sqrt{x}\cdot\frac{1}{x}dx$ $du=\frac{1}{x}dx$

$=2\sqrt{x}\ln x-2\int x^{-1/2}dx=\left(2\sqrt{x}\ln x-2\cdot 2\sqrt{x}\right)\Big|_4^9$

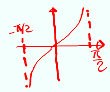
$=2\left[\sqrt{x}\ln x-2\sqrt{x}\right]\Big|_4^9=2\left[3\ln 3^2-6-2\ln 2^2+4\right]$

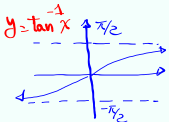
$=\boxed{2\left[6\ln 3-4\ln 2-2\right]}$

Jun 20-8:36 AM

Find the area bounded by $f(x)=\tan^{-1}\frac{1}{x}$, x-axis, $x=1$ and $x=\sqrt{3}$. Drawing Required.

$y=\tan x$

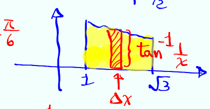




$\tan^{-1}\frac{1}{x} > 0$

$x=1 \quad \tan^{-1}1 = \frac{\pi}{4}$

$x=\sqrt{3} \quad \tan^{-1}\frac{1}{\sqrt{3}} = \tan^{-1}\frac{\sqrt{3}}{3} = \frac{\pi}{6}$



$$A = \int_1^{\sqrt{3}} \tan^{-1}\frac{1}{x}\,dx$$

$u=\tan^{-1}\frac{1}{x} \qquad dv = dx$

$$\frac{d}{dx}\left[\tan^{-1}u\right] = \frac{1}{1+u^2}\cdot\frac{du}{dx}$$

$du = \frac{1}{1+\left(\frac{1}{x}\right)^2}\cdot\frac{-1}{x^2}dx \qquad v = x$

$du = \frac{-1}{x^2+1}dx \qquad v=x$

$$= x\tan^{-1}\frac{1}{x} - \int \frac{-x}{x^2+1}dx$$

$$= x\tan^{-1}\frac{1}{x} + \frac{1}{2}\int\frac{2x}{x^2+1}dx = x\tan^{-1}\frac{1}{x} + \frac{1}{2}\ln(x^2+1)\Big|_1^{\sqrt{3}}$$

$$= \left(\sqrt{3}\tan^{-1}\frac{1}{\sqrt{3}} + \frac{1}{2}\ln 4\right) - \left(1\cdot\tan^{-1}1 + \frac{1}{2}\ln 2\right)$$

$$= \sqrt{3}\cdot\frac{\pi}{6} + \ln 2 - \frac{\pi}{4} - \frac{1}{2}\ln 2$$

$$= \boxed{\frac{\pi\sqrt{3}}{6} - \frac{\pi}{4} + \frac{1}{2}\ln 2}$$

Jun 20-8:49 AM

$$\int \cos\sqrt{x}\,dx$$

Hint: Make w-Subs., then use integration by Parts.

$w=\sqrt{x}$

$w^2 = x$

$2w\,dw = dx$

$$\int \cos w \cdot 2w\,dw$$

$u = w \qquad dv = \cos w\,dw$

$du = dw \qquad v = \sin w$

$$= 2\left[w\sin w - \int \sin w\,dw\right]$$

$$= 2w\sin w + 2\cos w + C$$

$$= 2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C$$

Jun 20-9:08 AM

Use the last hint to evaluate $\int_0^{\pi} e^{\cos x} \cdot \sin 2x \, dx$

Recall $\sin 2x = 2 \sin x \cos x$

$$\int_0^{\pi} e^{\cos x} \cdot 2 \sin x \cos x \, dx$$

$w = \cos x$

$dw = -\sin x \, dx$

$x = 0 \rightarrow w = 1$

$x = \pi \rightarrow w = -1$

$$\int_1^{-1} e^{w} \cdot 2 \cdot w \cdot -dw$$

$$= 2\int_{-1}^{1} w e^{w} dw$$

$u = w \qquad dv = e^{w} dw$

$du = dw \qquad v = e^{w}$

$$= 2\left[w e^{w} - \int e^{w} dw \right] = 2\left[w e^{w} - e^{w} \right]\Big|_{-1}^{1}$$

$$= 2\left[(\cancel{e^{1}} - \cancel{e^{1}}) - (-e^{-1} - e^{-1}) \right] = 2 \cdot 2e^{-1} = \boxed{\frac{4}{e}}$$

Jun 20-9:18 AM

7.2 Trignometric Integration

$$\int \sin^3 x \cos x \, dx$$

$u = \sin x$

$du = \cos x \, dx$

$$= \int u^3 \, du = \frac{u^4}{4} + C = \boxed{\frac{1}{4} \sin^4 x + C}$$

$$\int \sin^3 x \cos^5 x \, dx$$

Notice $\frac{d}{dx}[\sin x] = \cos x$

$\frac{d}{dx}[\cos x] = -\sin x$

$$= \int \sin^2 x \cdot \sin x \cdot \cos^5 x \, dx$$

$u = \cos x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$$= \int (1 - u^2) u^5 \cdot -du$$

$$= -\int (u^5 - u^7) du$$

$$= \frac{u^8}{8} - \frac{u^6}{6} + C = \frac{1}{8} \cos^8 x - \frac{1}{6} \cos^6 x + C$$

Jun 20-9:45 AM

$$\int \cos^2 x\,dx$$

Recall Double-Angle formula

$$\sin 2x = 2\sin x\cos x$$

$$\boxed{\cos 2x} = \cos^2 x - \sin^2 x$$
$$\boxed{= 2\cos^2 x - 1}$$
$$= 1 - 2\sin^2 x$$

Solve for $\cos^2 x$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\int \cos^2 x\,dx = \int \frac{1+\cos 2x}{2}\,dx$$
$$= \frac{1}{2}\int [1 + \cos 2x]\,dx$$
$$= \frac{1}{2}\left[x + \frac{1}{2}\sin 2x\right] + C$$
$$= \boxed{\frac{1}{2}x + \frac{1}{4}\sin 2x + C}$$

Jun 20-9:53 AM

$$\int \frac{\sin^2(\ln x)}{x}\,dx \qquad u = \ln x \qquad du = \frac{1}{x}\,dx$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$= \int \sin^2 u\,du$$
$$= \int \frac{1-\cos 2u}{2}\,du = \frac{1}{2}u - \frac{1}{2}\cdot\frac{1}{2}\sin 2u + C$$
$$= \frac{1}{2}\ln x - \frac{1}{4}\sin(2\ln x) + C$$
$$= \boxed{\ln\sqrt{x} - \frac{1}{4}\sin(\ln x^2) + C}$$

Jun 20-9:59 AM

$$\int x \sin^3 x \, dx$$

$u = x$ $\qquad$ $dv = \sin^3 x \, dx$

$dx = dx$ $\qquad$ $v = \int \sin^3 x \, dx$

$uv - \int v \, du$

$v = \int \sin^2 x \cdot \sin x \, dx$

$= x \cdot \left(\frac{1}{3}\cos^3 x - \cos x\right) - \int \left(\frac{1}{3}w^3 - w\right) dx$ $\qquad$ $= \int (1 - \cos^2 x) \sin x \, dx$

$= \frac{1}{3} x \cos^3 x - x\cos x - \int \left(\frac{1}{3}\cos^3 x - \cos x\right) dx$ $\qquad$ $= \int (1 - w^2) \cdot - dw$

$= \frac{1}{3} x \cos^3 x - x\cos x + \sin x - \frac{1}{3}\int \cos^3 x \, dx$ $\qquad$ $= \int (w^2 - 1) \, dw$

$\cos^3 x = \cos^2 x \cdot \cos x$ $\qquad$ $v = \frac{w^3}{3} - w$

$= (1 - \sin^2 x) \cdot \cos x$

Make Sure to finish

Jun 20-10:04 AM

$\int \tan x \, dx = \ln|\sec x| + C$ $\qquad$ $\tan x = \frac{\sin x}{\cos x}$

$u = \cos x$

$\int \sec x \, dx = \ln|\sec x + \tan x| + C$

$$\int \sec x \, dx = \int \frac{\sec x \cdot (\sec x + \tan x)}{\sec x + \tan x} dx$$

$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} dx$$

$= \int \frac{1}{u} du = \ln|u| + C$ $\qquad$ $u = \sec x + \tan x$

$du = (\sec^2 x + \sec x \tan x) dx$

$= \ln|\sec x + \tan x| + C$

$\frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\sec x] = \sec x \tan x$

$1 + \tan^2 x = \sec^2 x$

Jun 20-10:14 AM

$$\int \tan^5 x \, dx$$

$$= \int \tan^3 x \cdot \tan^2 x \, dx$$

$$= \int \tan^3 x \, (\sec^2 x - 1) \, dx = \int (\tan^3 x \sec^2 x - \tan^3 x) \, dx$$

$u = \tan x$

$du = \sec^2 x \, dx$

$$\int u^3 du - \int \tan x \cdot \tan^2 x \, dx$$

$$= \frac{u^4}{4} - \int \tan x \cdot (\sec^2 x - 1) \, dx = \frac{1}{4}\tan^4 x - \frac{1}{2}u^2 + \ln|\sec x| + C$$

$$= \frac{1}{4}\tan^4 x - \frac{1}{2}\tan^2 x + \ln|\sec x| + C$$

Jun 20-10:22 AM

$$\int x \sec x \tan x \, dx$$

$u = x \qquad dv = \sec x \tan x \, dx$

$du = dx \qquad v = \sec x$

$$uv - \int v \, du$$

$$= x \sec x - \int \sec x \, dx = x \sec x - \ln|\sec x + \tan x| + C$$

Jun 20-10:29 AM

$$\int \tan^3 x \sec^3 x \, dx$$

$$= \int \tan^2 x \cdot \sec^2 x \cdot \underbrace{\tan x \sec x \, dx}$$

$\tan^2 x \leftarrow \sec^2 x - 1$

$\int \tan x \sec x \, dx = \sec x + C$

$u = \sec x$

$du = \sec x \tan x \, dx$

$\frac{d}{dx}[\sec x] = \tan x \sec x$

$$= \int (u^2 - 1) \cdot u^2 \cdot du = \int (u^4 - u^2) du$$

$$= \frac{1}{5}u^5 - \frac{1}{3}u^3 + C$$

$$= \boxed{\frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C}$$

Jun 20-10:34 AM

$$\int \tan^3 x \sec^4 x \, dx = \int \tan^3 x \sec^2 x \cdot \sec^2 x \, dx$$

$\frac{d}{dx}[\tan x] = \sec^2 x$

$\sec^2 x = 1 + \tan^2 x$

Let $u = \tan x$

$du = \sec^2 x \, dx$

$$\int u^3 (1 + u^2) \, du$$

$$= \frac{1}{4}u^4 + \frac{1}{6}u^6 + C$$

$$= \frac{1}{4}\tan^4 x + \frac{1}{6}\tan^6 x + C$$

Jun 20-10:40 AM

$$\int \frac{\sin x + \cos x}{\sin 2x}\,dx = \int \frac{\sin x + \cos x}{2\sin x\cos x}\,dx$$

$$= \int \frac{\cancel{\sin x}}{2\cancel{\sin x}\cos x}\,dx + \int \frac{\cancel{\cos x}}{2\sin x\cancel{\cos x}}\,dx$$

$$= \frac{1}{2}\int \sec x\,dx + \frac{1}{2}\int \csc x\,dx$$

$$-\ln|\csc x + \cot x|$$

$$= \frac{1}{2}\ln|\sec x + \tan x| + \frac{1}{2}\cdot -\ln|\csc x + \cot x| + C$$

$$\frac{d}{dx}\left[-\ln|\csc x + \cot x|\right] = -1\cdot\frac{-\csc x\cot x - \csc^2 x}{\csc x + \cot x}$$

$$= -1\cdot\frac{-\csc x(\cancel{\cot x + \csc x})}{\cancel{\csc x + \cot x}}$$

$$= \csc x$$

Jun 20-10:45 AM

$$\int \frac{1}{\cos x - 1}\,dx = \int \frac{\cos x + 1}{(\cos x - 1)(\cos x + 1)}\,dx$$

$$= \int \frac{\cos x + 1}{\cos^2 x - 1}\,dx = \int \frac{\cos x + 1}{-\sin^2 x}\,dx$$

$$= -\int \frac{\cos x}{\sin^2 x}\,dx - \int \frac{1}{\sin^2 x}\,dx$$

$u = \sin x$

$du = \cos x\,dx$

$$= -\int \frac{1}{u^2}\,du \qquad - \int \csc^2 x\,dx$$

$$= -\cdot\frac{-1}{u} \qquad - \cdot -\cot x + C$$

$$= \boxed{\frac{1}{\sin x} + \cot x + C} \rightarrow = \boxed{\csc x + \cot x + C}$$

Jun 20-10:55 AM